

# Errata

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The following errata contains corrections to my article “Polyatomic Logic and Generalized Blok-Esakia” published by the Journal of Logic and Computation. I thank Antonio Cleani for pointing out the typo in the proof of Proposition 5.4, and to Simon Lemal for raising attention to the mistake in Proposition 5.9.

- (Pp.33 - Proof of Proposition 5.4, 2): The argument as presented is warped. What should have been written was that we want to prove the following:  $\text{Log}(\rho(\text{Var}(M))) \subseteq \rho(M)$ . Indeed assume that  $\varphi \in \text{Log}(\rho(\text{Var}(M)))$ . Suppose that  $B \models M$ . Then  $\rho(B) \models \varphi$ , so  $B \models T(\varphi)$ . Thus  $\varphi \in \rho(M)$ . Then by algebraic completeness  $\text{Var}(\rho(M)) \subseteq \text{Var}(\text{Log}(\rho(\text{Var}(M)))) = \rho(\text{Var}(M))$ .
- (Pp.31, Proposition 5.9) - The proof given that  $\tau$  is a complete homomorphism contains a gap which I have not been able to fix. Indeed, the proof of (3) given does not seem to follow, since the argument I have been able to find for it requires the full power of GMT Blok-Esakia. As such, Proposition 5.9 should be replaced by the following:

**Proposition 5.9.** Let  $\vdash_s \in \Lambda(\vdash_{\mathbf{X}})$  and  $\mathbf{K} \in \Xi(\mathbf{X})$ . Assume that  $\zeta$  is a strongly selective translation.

1.  $\tau(\mathbf{K})$  is a quasivariety.
2.  $\tau(\text{QVar}(\vdash_s)) = \text{QVar}(\tau(\vdash_s))$ . Hence  $\tau(\text{Log}(\mathbf{K})) = \text{Log}(\tau(\mathbf{K}))$ .
3.  $\tau$  is a complete meet-semilattice homomorphism.
4.  $\tau(\vdash_s)$  is the least  $\zeta$ -companion of  $\vdash_s$ .

The proof of Proposition 5.10 alludes to the previous proof mentioning that the case for  $\rho$  is similar. However, note that in fact  $\rho$  appears to be a complete homomorphism. First see that

$$\rho\left(\bigcap_{i \in I} K_i\right) = \bigcap_{i \in I} \rho(K_i)$$

To see it note that if  $A \in \rho(\bigcap_{i \in I} K_i)$  then there is some  $B \in \bigcap_{i \in I} K_i$  such that  $\theta(B) = A$  and so  $B \in K_i$  so  $A \in \bigcap_{i \in I} \rho(K_i)$ . On the other hand if  $A \in \bigcap_{i \in I} \rho(K_i)$ , then  $A = \theta(C_i)$  where each  $C_i \in K_i$ . Then  $\mathcal{F}(A) \leq C_i$  by sobriety, so  $\mathcal{F}(A) \in K_i$  for each  $i$ . Since  $\theta(\mathcal{F}(A)) \cong A$ , then  $A \in \rho(\bigcap_{i \in I} K_i)$ .

On the other hand we also have that:

$$\rho\left(\bigvee_{i \in I} K_i\right) = \bigvee_{i \in I} \rho(K_i)$$

Indeed, if  $A \in \rho(K_i)$ , then  $B \in K_i$  so  $B \in \bigvee_{i \in I} K_i$ , so  $A \in \rho(\bigvee_{i \in I} K_i)$ , which shows the right to left inclusion. On the other hand if  $A \in \rho(\bigvee_{i \in I} K_i)$ , then for some  $B \in \bigvee_{i \in I} K_i$ ,  $A = \theta(B)$ . Now  $B$  is a homomorphic image of  $C$  and  $C$  a subalgebra of  $D$ , and  $D = \prod_{i \in I} C_i$  where  $C_i \in K_i$ . Passing  $\theta$  through everything you obtain that  $\theta(B)$  will be the result of the constructions done in  $\bigvee_{i \in I} \rho(K_i)$ . This shows equality.

Proposition 5.9 is invoked in the key results of Section 6, namely the Polyatomic Blok-Esakia isomorphism. But in light of the previous fact, these results continue to work: we have that  $\tau$  and  $\rho$  are each others' inverses, and  $\rho$  is a complete homomorphism, hence, so is  $\tau$ .