

# Colimits of Heyting Algebras through Esakia Duality

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In contrast to Boolean algebras and Distributive lattices, the variety of Heyting algebras is not locally finite, and in fact, none of its free finitely generated algebras are finite. The associated difficulty of understanding the free algebras has motivated a wealth of research into describing such algebras. In this line study, Ghilardi constructed the free Heyting algebra generated by a finite distributive lattice, using Birkhoff duality. In this talk, I will present a recent generalization of this construction to the case of arbitrary distributive lattices, using Esakia duality. Categorically, this provides an explicit construction of a left adjoint to the inclusion of Heyting algebras in the category of distributive lattices. This has several applications, including the construction of colimits of Heyting algebras, as establishing new properties of the category of Heyting algebras, like co-distributivity. If time permits, I will also discuss some consequences of this for coalgebraic intuitionistic modal logic, and the relationship of positive and intuitionistic modal logic (this last part is joint work with Nick Bezhanishvili).