TOPICS IN ALGEBRAIC LOGIC AND DUALITY THEORY
SEMINAR SHEET 1

- The contents of this seminar sheet will be discussed on **June 04**;
- Pick one or more examples to work out in full detail; you do not need to work on all of the examples.
- Examples are roughly ordered by conceptual difficulty, relative to the material covered in lectures.

(1) **(Maksimova's equivalence)** Show the equivalence of Craig interpolation and superamalgamation.

(2) (**Amalgamation for varieties of modal algebras**)
Study interpolation in the following logics:
(a) **K**;
(b) $\mathbf{KD} = \mathbf{K} \oplus \Diamond\top$.
(c) $\mathbf{S4} = \mathbf{K} \oplus \Box p \to p \oplus \Box p \to \Box\Box p$.

(3) (**Amalgamation in Varieties of Heyting algebras**) Recall the variety **KC** of DeMorgan Heyting algebras; these are the Heyting algebras satisfying the *weak excluded middle* axiom:

$$\neg p \vee \neg\neg p$$

or equivalently, satisfying the DeMorgan law $\neg(a \wedge b) = \neg a \vee \neg b$.
(a) Show that an Esakia space X is dual to a DeMorgan Heyting algebra if and only if for each $x \in X$, there is a unique maximal point $y \in X$ such that $x \leq y$.
(b) Use the proof strategy for IPC to show that KC has interpolation.

(4) (**Locally finite amalgamation**) Suppose that $\mathcal{K}$ is a locally finite variety (of Heyting or modal algebras). Show that $\mathcal{K}$ has amalgamation if and only if each triple (A, B_1, B_2, f_1, f_2) where A, B_1, B_2 are finite subdirectly irreducible algebras in the variety, has an amalgamation in the variety.
Use this criterion to conclude that the logics **LC** and $\mathbf{BD}_2$ have interpolation.