Introduction to Topology in and via Logic
Kick-Off Meeting

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January 4, 2023

Introduction Round

Practicalities and Organization

- Six lectures on core concepts of general topology
  - Given by Rodrigo and Søren.
  - On Tuesdays, Wednesdays and Fridays of first two weeks (STC: still waiting for confirmation on lecture rooms).
  - Hybrid and will be recorded.
- Group presentations on more advanced topics
  - Given by you, the students.
  - On Tuesdays, Wednesdays and Fridays of fourth week.
  - Aimed at your fellow students.
- Two 'large' HW assignments
  - Similar to HWs in courses like MPML or IML.
  - Published on Jan 10 and Jan 17, respectively.
  - Deadline is Feb 10.
  - Done in teams.
- Three small online multiple-choice quizzes
  - As a way of engaging in covered material and as a check (for you) on whether you've understood definitions and ideas.
  - Published on Tuesdays and Fridays.
  - Deadlines before Friday and Tuesday lectures.
  - Done individually.

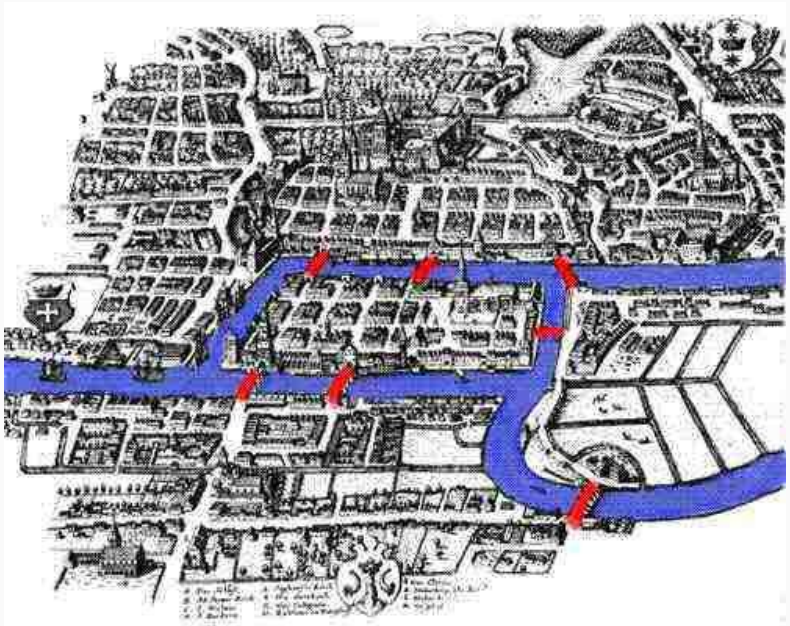
Schedule

Week 0	• Jan 4, Thurs, 11am: Kick-off meeting
Week 1	• Jan 9, Mon, 11am: Lecture notes (WIP) published
	• *Jan 10, Tues, 11am: 1st lecture (by Søren)
	• Jan 10, Tues: 1st assignment published (covering W1 material)
	• *Jan 11, Wed, 11am: 2nd lecture (by Søren)
	• Jan 13, Fri, 10.30am: 1st MC quiz
	• *Jan 13, Fri, 11am: 3rd lecture (by Søren)
Week 2	• Jan 17, Tues, 10.30am: 2nd MC quiz
	• *Jan 17, Tues, 11am: 4th lecture (by Rodrigo)
	• Jan 17, Tues: 2nd assignment published (covering W2 material)
	• *Jan 18, Wed, 11am: 5th lecture (by Rodrigo)
	• Jan 20, Fri, 10.30am: 3rd MC quiz
	• *Jan 20, Fri, 11am: 6th lecture (by Rodrigo)
Week 3	• *Jan 24, Tues, 11am: Guest lecture
	• Team consultations
Week 4	• *Jan 31, Tues, 11am: Team presentations (by you, in teams)
	• *Feb 1st, Wed, 11am: Team presentations (by you, in teams)
	• *Feb 3rd, Fri, 11am: Team presentations (by you, in teams)
Week 5	• Feb 10: Deadline for assignments

* STC, waiting for confirmation.

What is Topology About: A Bit of History

A long time ago:



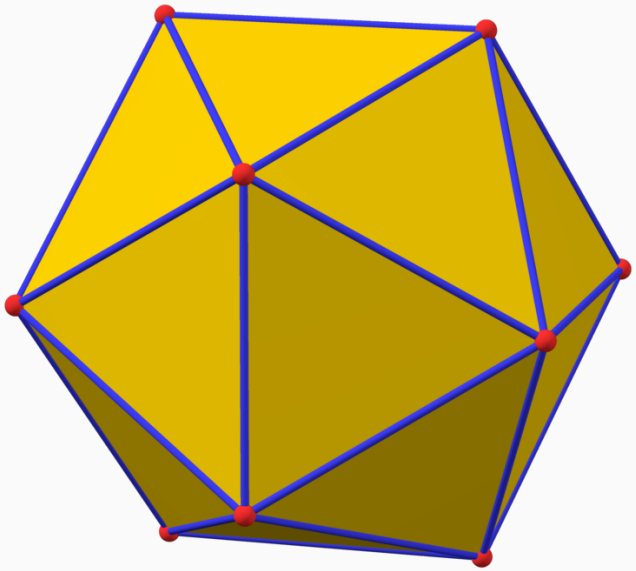
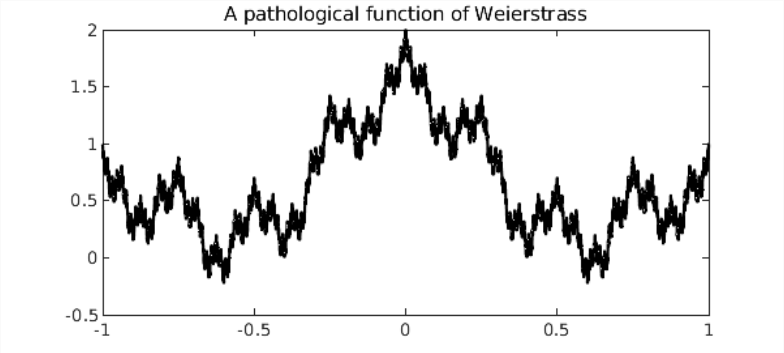


Figure 1: The Humble Origins of Topology

What is Topology About: A Bit of History

But then things got wild:



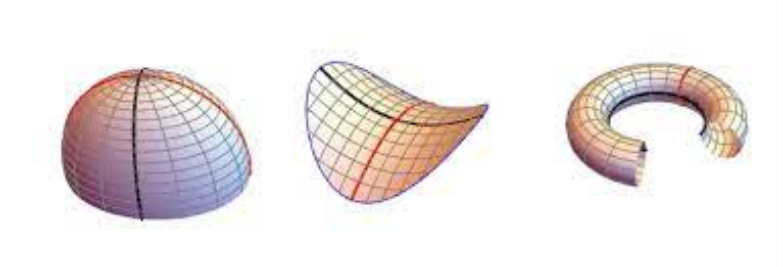
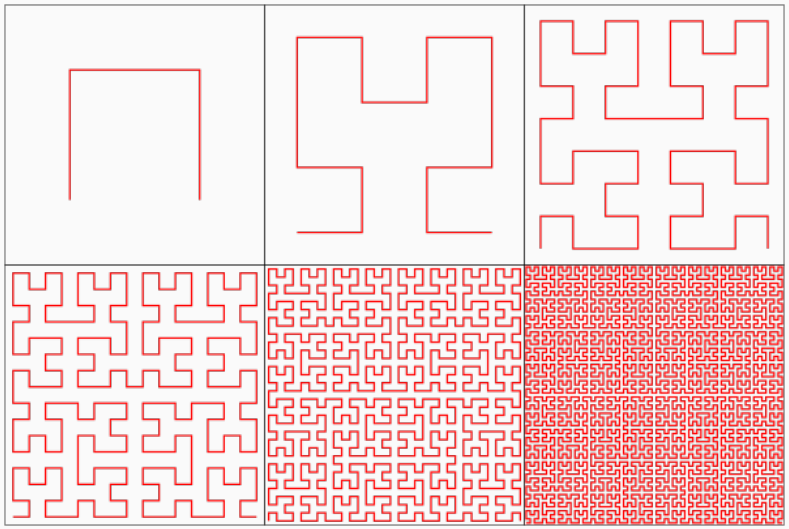


Figure 2: ????????



What is Topology About: A Bit of History

It was necessary to develop a general theory. This was done in stages.

The eventual formalism that was settled upon very far removed from our geometric intuitions. But this opens room for different intuitions: epistemic intuitions.

Our approach will try to emphasise the epistemic angle as a way to bootstrap our understanding, but we will also discuss the geometric approach.

What is Topology: Epistemic Intuitions

A space is a pair (X, τ) where $\tau \subseteq \mathcal{P}(X)$ is a collection of *possibly knowable facts*. These are facts which can be verified – there is some procedure to generate them. Their complements are falsifiable statements.

Not all facts are knowable (e.g. what is the precise velocity and position of a particle in a quantum field). Not all facts are falsifiable (e.g. famously, Popper said this of a few infamous 20th century philosophical ideas).

We want to think of τ as the "recursively enumerable" facts; those $A \in \tau$ such that $X - A \in \tau$ we say are decidable facts.

What concepts we will cover, and their Epistemic/Geometric Meaning

1. Topological Spaces: collections of epistemic worlds; generalized spaces;
2. Continuity: Transforming epistemic settings; transforming spaces;
3. Separation: Being able to tell apart two worlds using a proposition; being able to distinguish points using opens;
4. Compactness: Being able to describe all worlds using finite amounts of information; having the space be appropriately "complete".
5. Connectedness: Disconnectedness is epistemically the capacity to have enough "decidable" propositions; connectedness is geometrically having the space be accessible through paths (or something close enough).

Thank you!
Questions?